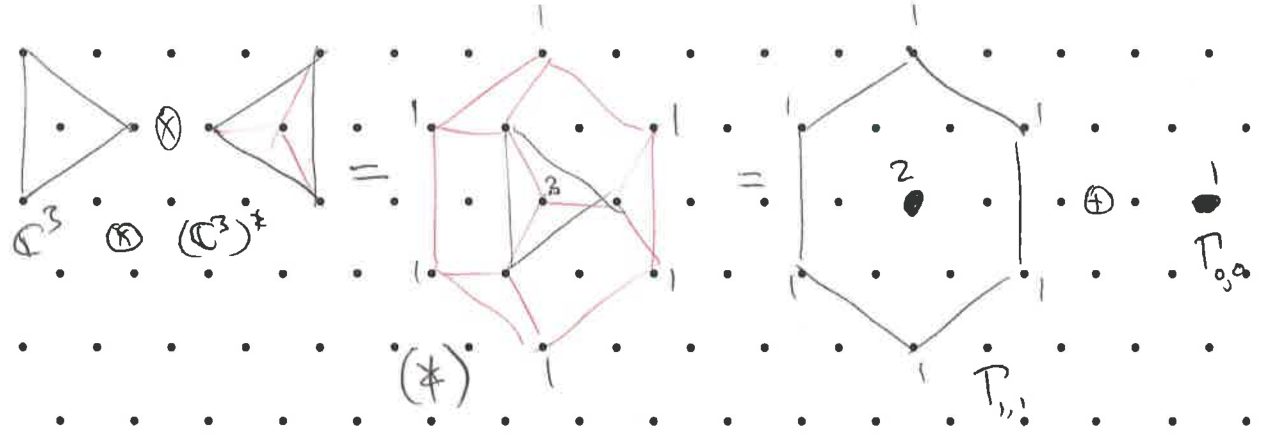
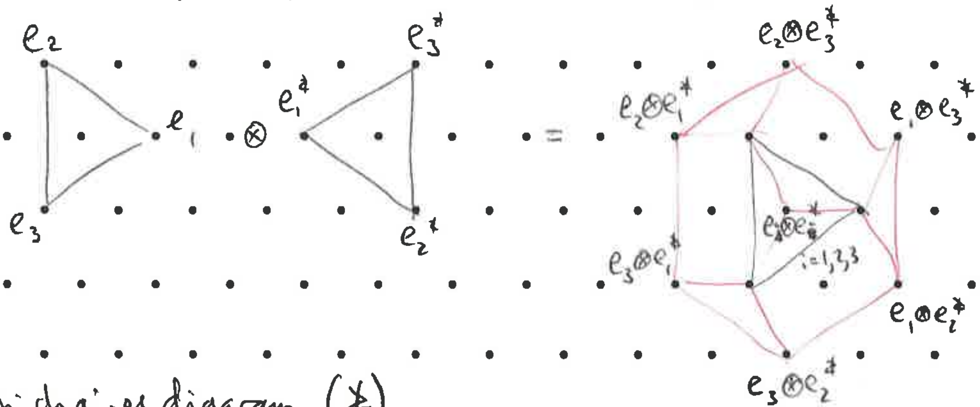
3.c)



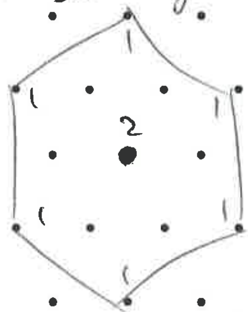
To find the weight diagram of $\mathbb{C}^3 \otimes (\mathbb{C}^3)^*$ remember that weights add, so:



Which gives diagram (∗).

Now strip off irreducible subreps starting with the one generated by the highest weight vector (in this case $e_1 \otimes e_3^*$ with weight $L_1 - L_3$). This generates a copy of $\Gamma_{1,1}$)

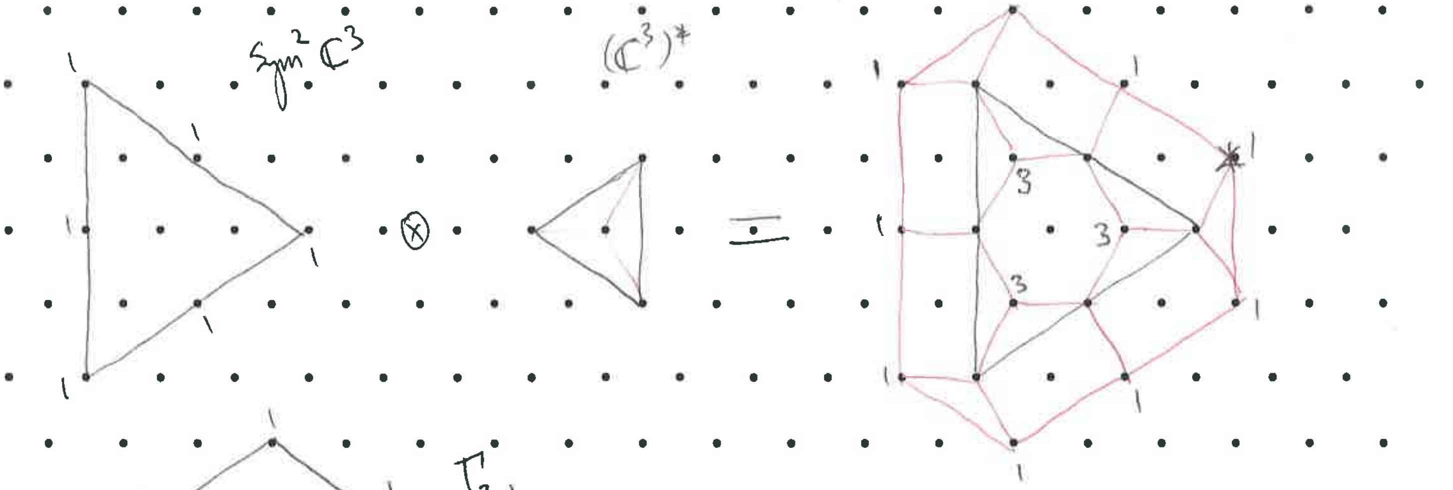
Since $\Gamma_{1,1}$ has weight diagram

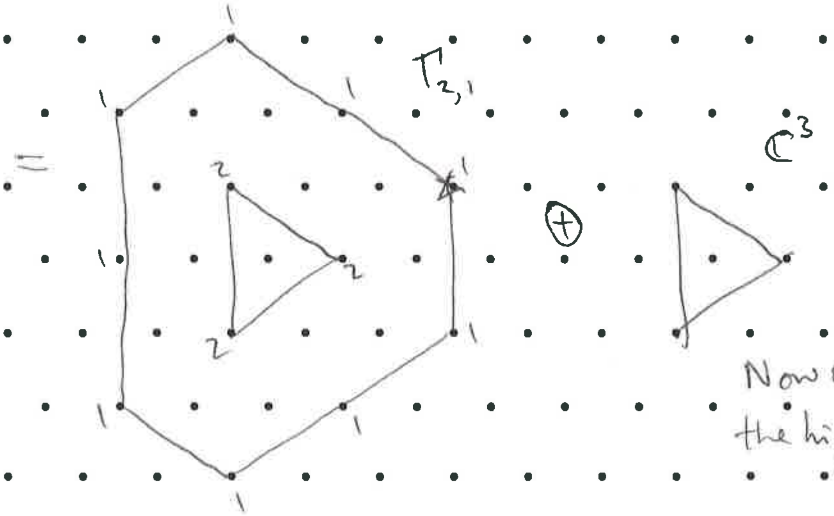


all that is left is a trivial one-dim. rep with weight 0. This is actually spanned by $\sum_{i=1}^{3} e_i \otimes e_i^*$.

3. d)

To find the weights of the tensor product add the weight diagrams by superimposing the diagram of $(\mathbb{C}^3)^*$ onto every vertex of $\mathrm{Sym}^2\mathbb{C}^3$ and adding weights.

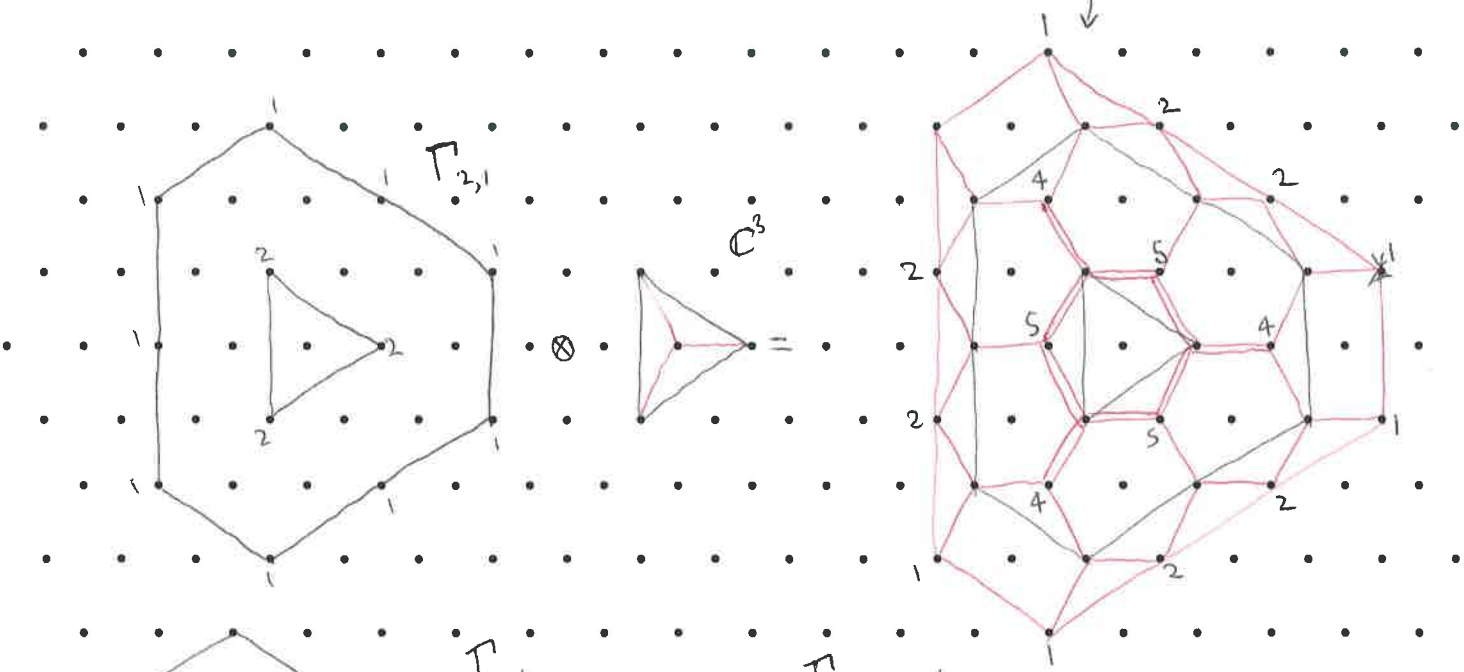


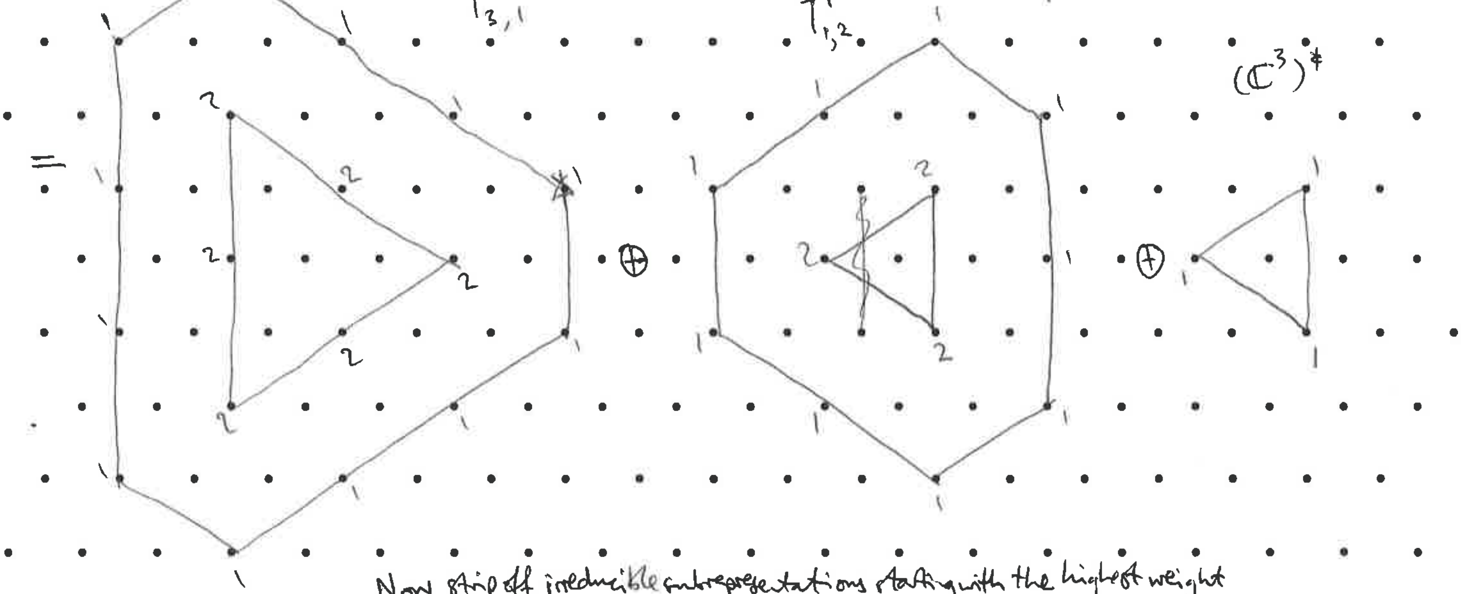


Now strip off subreps starting with the highest weight (in this case $2L_1 - L_3$ marked with a $*$).

3. e)

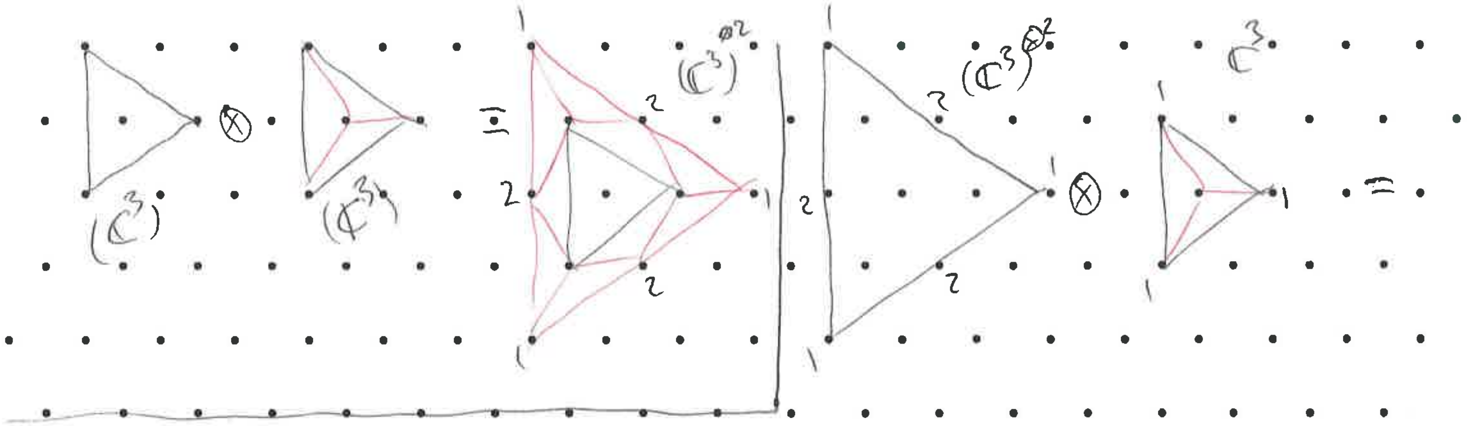
To find the weights of the tensor product, add the weight diagrams by superimposing the diagram for $\mathbb{C}^3$ onto every vertex in $\Gamma_{2,1}$ and adding multiplicities

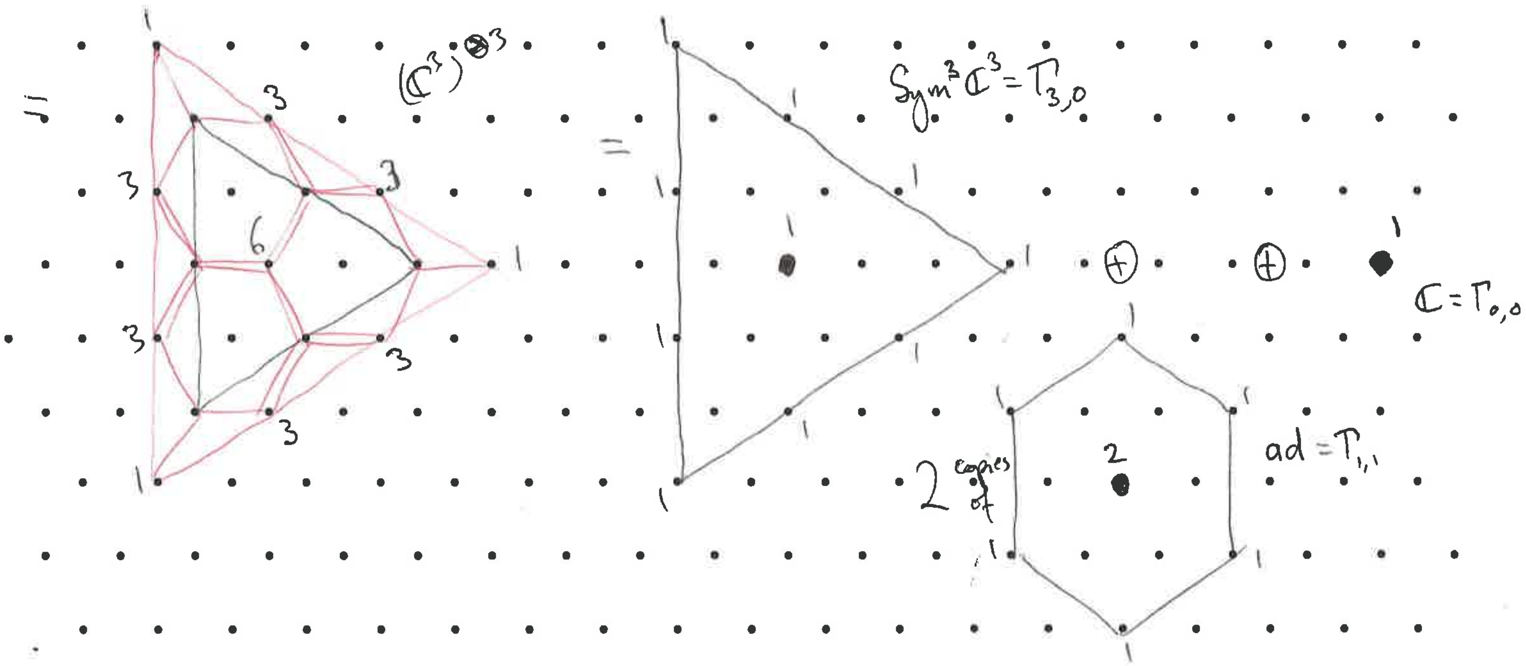




Now strip off irreducible subrepresentations starting with the highest weight (in this case $3L_1 - L_3$, marked with a $*$).

3. f)


$(\mathbb{C}^3)$ ⊗ $(\mathbb{C}^3)$ = $(\mathbb{C}^3)^{\otimes 2}$
$(\mathbb{C}^3)^{\otimes 2}$ ⊗ $\mathbb{C}^3$ =


$(\mathbb{C}^3)^{\otimes 3}$
$\mathrm{Sym}^3\mathbb{C}^3 = \Gamma_{3,0}$
$\mathbb{C} = \Gamma_{0,0}$
2 copies of
$\mathrm{ad} = \Gamma_{1,1}$